Standard Deviation

Lecture 18
Section 5.3.4

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Outline

Exercise 5.12, page 333.

The five-number summary for the distribution of income (in \$1000s) for the 200 households in your neighborhood is provided below.

$$\$25, \$37, \$67, \$100, \$250$$

Exercise 5.12, page 333.

(a) Draw a basic boxplot for the income distribution in your neighborhood.

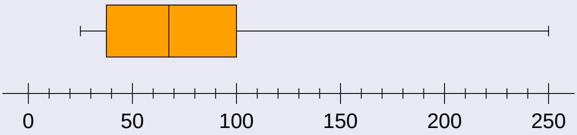
(b) Suppose that your household income is \$56,000. What can you say about the percentage of households that have a higher income than you?

(c) If the lowest 25% of the households will be classified as poor, what is the minimum household income that would lead to being classified as not poor?

Solution

(a)

- First, do not find a five-number summary for these data. These numbers are the five-number summary.
- The boxplot:



Solution

(b) \$56,000 is between the first quartile and the median, so we can say that at least half the neighborhood, but no more than three-quarters, have a higher income.

(c) You must have an income of at least \$37,000 not to be classified as poor.

Variability

- Our ability to estimate a parameter accurately depends on the variability of the population.
- What do we mean by "variability" in the population?
- How do we measure it?

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Deviations from the Mean

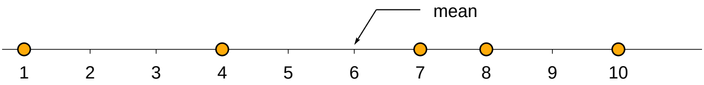
Definition (Deviation)

The deviation of an observation x is the difference between x and the sample mean $\bar{x}$.

$$\text{deviation of } x = x - \bar{x}.$$

Deviations from the Mean

- Deviations from the mean.



Deviations from the Mean

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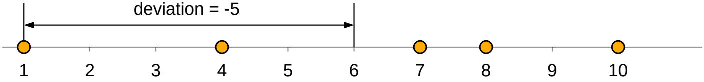

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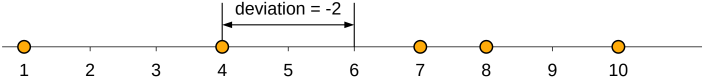

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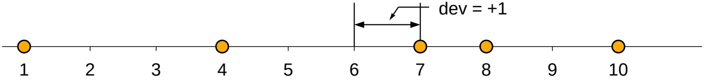

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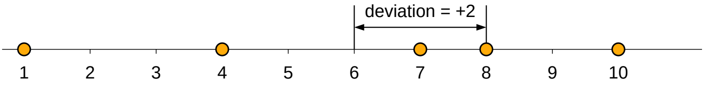

Deviations from the Mean

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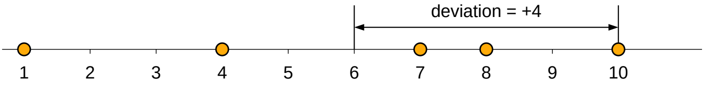


Deviations from the Mean

- How do we obtain one number that is representative of the whole set of individual deviations?
- Normally we use an average to summarize a set of numbers.
- Why will the average not work in this case?

Sum of Squared Deviations

- Rather than average the deviations, we will average their squares. That way, there will be no canceling.
- So we compute the sum of the squared deviations.

Definition (Sum of squared deviations)

The sum of squared deviations, denoted SSX, of a set of numbers is the sum of the squares of their deviations from the mean of the set.

$$\text{SSX} = \sum (x - \overline{x})^2.$$

Sum of Squared Deviations

To find SSX

- Find the average: $\bar{x} = \frac{\sum x}{n}$.
- Find the deviations from the average: $x - \bar{x}$.
- Square the deviations: $(x - \bar{x})^2$.
- Add them up:

$$\text{SSX} = \sum (x - \bar{x})^2.$$

Sum of Squared Deviations

Example (Calculating SSX)

- Let the sample be $\{1, 4, 7, 8, 10\}$.
- Then

$$\begin{aligned} \text{SSX} &= (1-6)^2 + (4-6)^2 + (7-6)^2 \\ &\quad +(8-6)^2 + (10-6)^2 \\ &= (-5)^2 + (-2)^2 + (1)^2 + (2)^2 + (4)^2 \\ &= 25 + 4 + 1 + 4 + 16 \\ &= 50. \end{aligned}$$

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 3, 4, 6, 9, 11, 15\}$.
- Calculate
  - The sample mean.
  - The deviations.
  - The squared deviations.
  - The sum of the squared deviations.

The Population Variance

Definition (Variance of a population)

The variance of a population, denoted σ^2, is the average of the squared deviations of the members of the population.

$$\sigma^2 = \frac{\sum (x-\mu)^2}{N}.$$

Definition (Standard deviation of a population)

The standard deviation of a population, denoted σ, is the square root of the population variance.

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}.$$

The Sample Variance

Definition (Variance of a sample)

The variance of a sample, denoted s^2, is the sum of the squared deviations of the members of the sample, divided by 1 less than the sample size.

$$s^2 = \frac{\sum (x - \overline{x})^2}{n - 1}.$$

Definition (Standard deviation of a sample)

The standard deviation of a sample, denoted s, is the square root of the sample variance.

$$s = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}.$$

The Sample Variance

- Theory shows that if we divide $\sum (x - \overline{x})^2$ by $n - 1$ instead of n, then s^2 will be a better estimator of σ^2.
- Otherwise, s^2 will systematically underestimate σ^2.
- Therefore, we do it.

Example

Example (Calculating s)

- For the sample $\{1, 4, 7, 8, 10\}$, we found that

$$\text{SSX} = 50.$$

- Therefore,

$$s^2 = \frac{50}{4} = 12.5$$

and so

$$s = \sqrt{12.5} = 3.536.$$

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 3, 4, 6, 9, 11, 15\}$.
- Calculate s^2 and s.

Example

- How does s compare to the individual deviations?
- We will interpret s as being representative of the deviations in the sample.
- Does that seem reasonable for the previous examples?

Alternate Formula for SSX

- An alternate formula to compute SSX is

$$\text{SSX} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}.$$

- Then, as before

$$s^2 = \frac{\text{SSX}}{n-1}$$

and

$$s = \sqrt{\frac{\text{SSX}}{n-1}}.$$

Example

Example (Alternate formula for SSX)

- Let the sample be $\{1, 4, 7, 8, 10\}$.
- Then

$$\sum x = 30$$

and

$$\sum x^2 = 1 + 16 + 49 + 64 + 100 = 230.$$

- So

$$\begin{aligned} \text{SSX} &= 230 - \frac{30^2}{5} \\ &= 230 - 180 \\ &= 50. \end{aligned}$$

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 3, 4, 6, 9, 11, 15\}$.
- Find $\sum x$.
- Find $\sum x^2$.
- Use the alternate formula to find SSC, s^2, and s.

TI-83 - Standard Deviations

TI-83 Standard Deviations

- Follow the procedure for computing the mean.
- The display shows `Sx` and σ`x`.
- `Sx` is the sample standard deviation.
- σ`x` is the population standard deviation.

Example

Example

TI-83 Standard Deviations

- Let the sample be $\{1, 4, 7, 8, 10\}$.
- We get
  - $\mathrm{Sx} = 3.535533906$.
  - $\sigma\mathrm{x} = 3.16227766$.

Sum of Squared Deviations

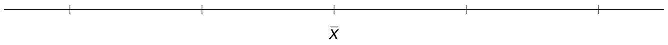
Practice

- Let the sample be $\{1, 3, 4, 6, 9, 11, 15\}$.
- Use the TI-83 to find s and s^2.
- What are the values of $\sum x$ and $\sum x^2$?

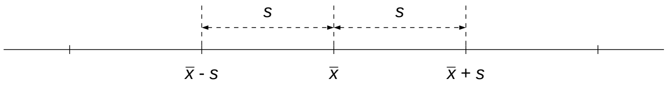
Interpreting the Standard Deviation

- Observations that deviate from $\bar{x}$ by much more than s are unusually far from the mean.
- Observations that deviate from $\bar{x}$ by much less than s are unusually close to the mean.

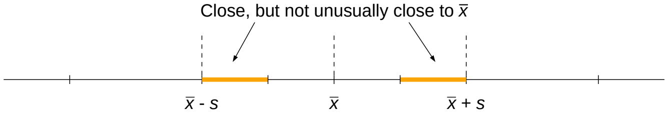
Interpreting the Standard Deviation



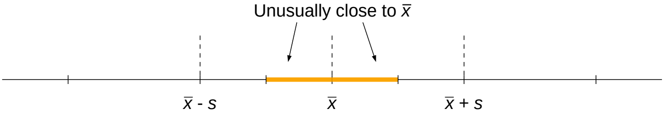
Interpreting the Standard Deviation



Interpreting the Standard Deviation



Interpreting the Standard Deviation



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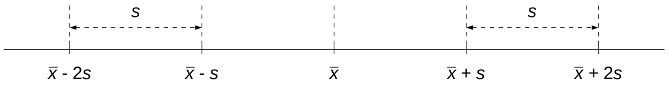

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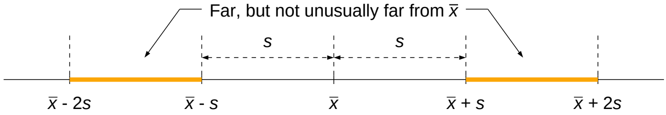
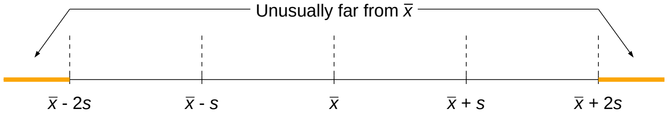


Interpreting the Standard Deviation



Assignment

Homework

- Read Section 5.3.4, pages 326 - 333.
- Let's Do It! 5.13, 5.14, 5.15.
- Page 333, exercises 10, 11, 14, 16 - 18, 20, 21.
- Chapter 5 review, p. 345, exercises 29 - 32, 36 - 40, 42 - 44, 47, 52, 53, 55.